

Search for excited spin-3/2 neutrinos at LHeC

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Abstract

We study the potential of the next ep collider, namely LHeC, with two options $\sqrt{s} = 1.3$ TeV $\sqrt{s} = 1.98$ TeV, to search for excited spin-1/2 and spin-3/2 neutrinos. We calculate the single production cross section of excited spin-1/2 and spin-3/2 neutrinos according to their effective currents describing their interactions between gauge bosons and SM leptons. We choose the $\nu^* \rightarrow eW$ decay mode of excited neutrinos and $W \rightarrow jj$ decay mode of W -boson for the analysis. We put some kinematical cuts for the final state detectable particles and plot the invariant mass distributions for signal and the corresponding backgrounds. In order to obtain accessible limits for excited neutrino couplings, we show the $f - f'$ and $c_{iV} - c_{iA}$ contour plots for excited spin-1/2 and excited spin-3/2 neutrinos, respectively.

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I. INTRODUCTION

The Standard Model (SM) of the particle physics is in accordance with the experimental outcomes received from the operating colliders. The first run of the Large Hadron Collider (LHC) brought the expected Higgs boson discovery, so a crucial part of the SM had been completed. But there is still no satisfying answer about the three-family structure of leptons and quarks and mass hierarchy of them. An attractive explanation is lepton and quark compositeness [1]. In composite models, known leptons and quarks have a substructure characterized by an energy scale named compositeness scale, Λ . A natural consequence of compositeness is the occurrence of excited states [2, 3]. Phenomenologically, an excited lepton can be regarded as a heavy lepton sharing the same leptonic quantum number with the corresponding SM lepton. If leptons present composite structures, they can be considered as spin-1/2 bound states containing three spin-1/2 or spin-1/2 and spin-0 subparticles. Bound states of spin-3/2 leptons also possible with three spin-1/2 [1] or spin-1/2 and spin-1 subparticles in the framework of composite models [4]. The motivations for spin-3/2 particles come from two different scenarios; spin-3/2 leptons appear in composite models [5, 6]; and a spin-3/2 gravitino is the superpartner of graviton in supergravity[7]. One can find some of the latest studies about beyond the Standard Model theories including exotic spin-3/2 particles in [8].

Both excited spin-1/2 and spin-3/2 neutrinos can be produced at future high energy lepton, hadron and lepton-hadron colliders. Elaborate studies on excited spin-1/2 neutrinos can be found in [9–13]. Also, one can find excited spin-1/2 neutrino production by ultra high energy neutrinos in [14] and the impact of excited spin-1/2 neutrinos on $\nu\bar{\nu} \rightarrow \gamma\gamma$ process in [15].

The mass limit for excited spin-1/2 neutrinos obtained from their pair production ($e^+e^- \rightarrow \nu^*\nu^*$ process) by L3 Collaboration at $\sqrt{s} = 189 - 209$ GeV, assuming $f = -f'$, where f and f' are the new couplings determined by the composite dynamics, is $m^* > 102.6$ GeV [16]. Assuming $f = f'$ and $f/\Lambda = 1/m^*$, for single production of excited spin-1/2 neutrino in ep collisions taking into account all the decay channels, the H1 Collaboration set the exclusion limit for the mass range of excited neutrino $m^* > 213$ GeV at 95% C.L.[17]. Recently, a search was performed by the ATLAS Collaboration taking into account pair production of excited spin-1/2 neutrinos either through contact or gauge-mediated interactions

and their decay proceeds via the same mechanism. Considering events with at least three charged leptons with $\Lambda = m^*$, $f = f' = 1$ and with an integrated luminosity of 20.3 fb^{-1} of pp collisions at $\sqrt{s} = 8 \text{ TeV}$; lower mass limit obtained as 1.6 TeV for every excited spin-1/2 neutrino flavour[18].

Excited spin-3/2 neutrinos were least studied in the literature by the side of the spin-1/2 ones. An investigation for the production and decay process of the single heavy spin-3/2 neutrino was performed in [19, 20]. The study for the potential of future high energy e^+e^- linear colliders to probe excited spin-3/2 neutrino signals in different decay modes in the frame of three phenomenological currents taking into account the corresponding background was done in [4].

Studies are ongoing for the development of a new ep collider, the Large Hadron Electron Collider (LHeC), with an electron beam of 60 GeV , to possibly 140 GeV , and a proton beam of the LHC [21] or in the future the Future Circular Collider lepton-hadron collider (FCC-eh)[22]. The LHeC is the highest energy lepton-hadron collider under design and is considered as a linac-ring collider. Linac-ring type colliders were proposed in [23] and, the physics potentials and advantages of these type lepton-hadron colliders are discussed in [24]. Latest results for excited neutrino searches coming from the first ep collider HERA have showed that ep colliders are so competitive to pp and e^+e^- colliders and very important for the investigation of beyond SM physics [17, 21]. With the design luminosity of $10^{33} \text{ cm}^{-2}\text{s}^{-1}$ the LHeC is intended to exceed the HERA luminosity by a factor of ~ 100 . So it would be a major opportunity to push forward the investigations done in the LHC.

This work is a continuation of the previous works on excited neutrinos [4, 11]. In this work, in Section II we introduce the phenomenological currents for excited neutrinos and give the decay widths of them. In Section III, we consider single production of excited spin-1/2 and spin-3/2 neutrinos at ep colliders. We take into account the signal in $\nu^* \rightarrow eW$ decay mode of excited neutrinos as well as corresponding backgrounds at LHeC with $\sqrt{s} = 1.3 \text{ TeV}$ and $\sqrt{s} = 1.98 \text{ TeV}$. We plot the invariant mass distributions for single production of excited neutrinos with spin-1/2 and spin-3/2. Last, we plot the contour plots for the excited neutrino couplings to obtain the exclusion limits. Investigation on excited fermions with spin-1/2 take an important part in the physics program of LHeC. Although the latest limit for excited spin-1/2 neutrinos set by the ATLAS experiment is high, it is important to examine the excited neutrinos with different spins at high energy lepton-hadron colliders.

This work is the only dedicated work which gives the comparative results both for excited spin-1/2 and spin-3/2 neutrinos to comprehend the potential of next ep collider.

II. PHYSICAL PRELIMINARIES

An excited spin-1/2 neutrino is the lowest radial and orbital excitation according to the classification by $SU(2) \times U(1)$ quantum numbers. Interactions between excited spin-1/2 neutrino and ordinary leptons are magnetic transition type [25–27]. The effective current for the interaction between an excited spin-1/2 neutrino, a gauge boson ($V = \gamma, Z, W^\pm$), and the SM lepton is given by

$$J^\mu(1/2) = \frac{g_e}{2\Lambda} \bar{u}(k, 1/2) i\sigma^{\mu\nu} q_\nu (1 - \gamma_5) f_V u(p, 1/2) \quad (1)$$

where Λ is the new physics scale; g_e is electromagnetic coupling constant with $g_e = \sqrt{4\pi\alpha}$; k, p and q are the four momentum of the SM lepton, excited spin-1/2 neutrino and the gauge boson, respectively. f_V is the new electroweak coupling parameter corresponding to the gauge boson V and $\sigma^{\mu\nu} = i(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)/2$ with γ^μ being the Dirac matrices. An excited neutrino has three possible decay modes each of one is related to a vector boson γ, W and Z ; radiative decay $\nu^* \rightarrow \nu\gamma$, neutral weak decay $\nu^* \rightarrow \nu Z$, and charged weak decay $\nu^* \rightarrow eW$. Neglecting SM lepton mass we find the decay width of excited spin-1/2 neutrino as

$$\Gamma(l^{*(1/2)} \rightarrow lV) = \frac{\alpha m^{*3}}{4\Lambda^2} f_V^2 \left(1 - \frac{m_V^2}{m^{*2}}\right)^2 \left(1 + \frac{m_V^2}{2m^{*2}}\right) \quad (2)$$

where $f_\gamma = (f - f')/2, f_Z = (f \cot\theta_W + f' \tan\theta_W)/2, f_W = f/\sqrt{2} \sin\theta_W$; θ_W is the weak mixing angle and m_V is the mass of the gauge boson. The couplings f and f' are the scaling factors for the gauge couplings of $SU(2)$ and $U(1)$. It is remarkable that for the choice of the couplings $f = -f'$, the electromagnetic interaction of excited neutrino and SM neutrino exists. Branching ratios of excited spin-1/2 neutrino are presented in Table I. One may note that for the choice $f = -f' = 1$ the branching ratio for the eW channel is 60%. Hence, to choose the $\nu^* \rightarrow eW$ mode for the analysis is more feasible.

Table I: Brancing ratios and total decay width of excited spin-1/2 neutrinos for $f = -f' = 1$ ($f = f' = 1$) Here it is taken $\Lambda = m^*$.

$m^*(GeV)$	$\Gamma(GeV)$	$\%BR(\nu^* \rightarrow \nu\gamma)$	$\%BR(\nu^* \rightarrow \nu Z)$	$\%BR(\nu^* \rightarrow eW)$
300	1.91	30.5 (0)	10.7 (38.3)	58.9 (61.7)
500	3.36	28.9 (0)	11.1 (38.9)	60.0 (61.1)
750	5.12	28.4 (0)	11.3 (39.0)	60.3 (61.0)
1000	6.87	28.2 (0)	11.3 (39.1)	60.4 (60.9)
1500	10.35	28.1 (0)	11.4 (39.1)	60.5 (60.9)
2000	13.82	28.1 (0)	11.4 (39.1)	60.5 (60.9)
2500	17.28	28.1 (0)	11.4 (39.1)	60.5 (60.9)
3000	20.75	28.1 (0)	11.4 (39.1)	60.5 (60.9)

The two phenomenological currents for the interactions between an excited spin-3/2 neutrino, a gauge boson ($V = \gamma, Z, W^\pm$), and the SM lepton are given by

$$J_1^\mu(3/2) = g_e \bar{u}(k, 1/2)(c_{1V} - c_{1A}\gamma_5)u^\mu(p, 3/2), \quad (3)$$

$$J_2^\mu(3/2) = \frac{g_e}{\Lambda} \bar{u}(k, 1/2)q_\lambda \gamma^\mu (c_{2V} - c_{2A}\gamma_5)u^\lambda(p, 3/2), \quad (4)$$

Decay widths of excited spin-3/2 neutrinos for the $\nu^* \rightarrow \nu\gamma$ decay mode for the two currents are given by

$$\Gamma_1(\nu^{*(3/2)} \rightarrow \nu\gamma) = \frac{\alpha}{4}(c_{1v}^2 + c_{1A}^2)m^*, \quad (5)$$

$$\Gamma_2(\nu^{*(3/2)} \rightarrow \nu\gamma) = \frac{\alpha}{24}(c_{2v}^2 + c_{2A}^2)m^*\left(\frac{m^*}{\Lambda}\right)^2, \quad (6)$$

and for the neutral and charged weak decay modes ($\nu^* \rightarrow \nu Z$ and $\nu^* \rightarrow eW$) given as

$$\Gamma_1(\nu^{*(3/2)} \rightarrow lV) = \frac{\alpha}{48}(c_{1v}^2 + c_{1A}^2)m^*\frac{(1-\kappa)^2}{\kappa}(1+10\kappa+\kappa^2), \quad (7)$$

$$\Gamma_2(\nu^{*(3/2)} \rightarrow lV) = \frac{\alpha}{48}(c_{2v}^2 + c_{2A}^2)m^*\left(\frac{m^*}{\Lambda}\right)^2\frac{(1-\kappa)^4}{\kappa}(1+2\kappa), \quad (8)$$

where $\kappa = (m_V/m_\star)^2$, $V = Z, W$, and $l = e, \nu$. Branching ratios and total decay width of excited spin-3/2 neutrinos with J_1 and J_2 are given in Table II and Table III, respectively. Also, total decay width of excited neutrinos as a function of their mass (m^\star) is shown in Figure 1.

Table II: Branching ratios and total decay width of excited spin-3/2 neutrinos with J_1 . Here it is taken $c_{1V} = c_{1A}=0.5$ and $\Lambda = m^\star$.

$m^\star(GeV)$	$\Gamma(GeV)$	$\%BR(\nu^\star \rightarrow \nu\gamma)$	$\%BR(\nu^\star \rightarrow \nu Z)$	$\%BR(\nu^\star \rightarrow eW)$
300	1.21	24.0	34.4	41.6
500	3.89	12.5	39.0	48.5
750	11.11	6.5	41.2	52.3
1000	24.61	3.9	42.1	54.0
1500	78.89	1.8	42.8	55.3
2000	183.50	1.1	43.1	55.9
2500	355.20	0.7	43.2	56.1
3000	611.00	0.5	43.3	56.2

Table III: Branching ratios and total decay width of excited spin-3/2 neutrinos with J_2 . Here it is taken $c_{2V} = c_{2A}=0.5$ and $\Lambda = m^\star$.

$m^\star(GeV)$	$\Gamma(GeV)$	$\%BR(\nu^\star \rightarrow \nu\gamma)$	$\%BR(\nu^\star \rightarrow \nu Z)$	$\%BR(\nu^\star \rightarrow eW)$
300	0.55	8.8	38.4	52.8
500	2.71	3.0	41.8	55.3
750	9.31	1.3	42.7	56.0
1000	22.21	0.7	43.0	56.2
1500	75.26	0.3	43.3	56.4
2000	178.7	0.2	43.4	56.5
2500	349.2	0.1	43.4	56.5
3000	603.6	0.1	43.4	56.5

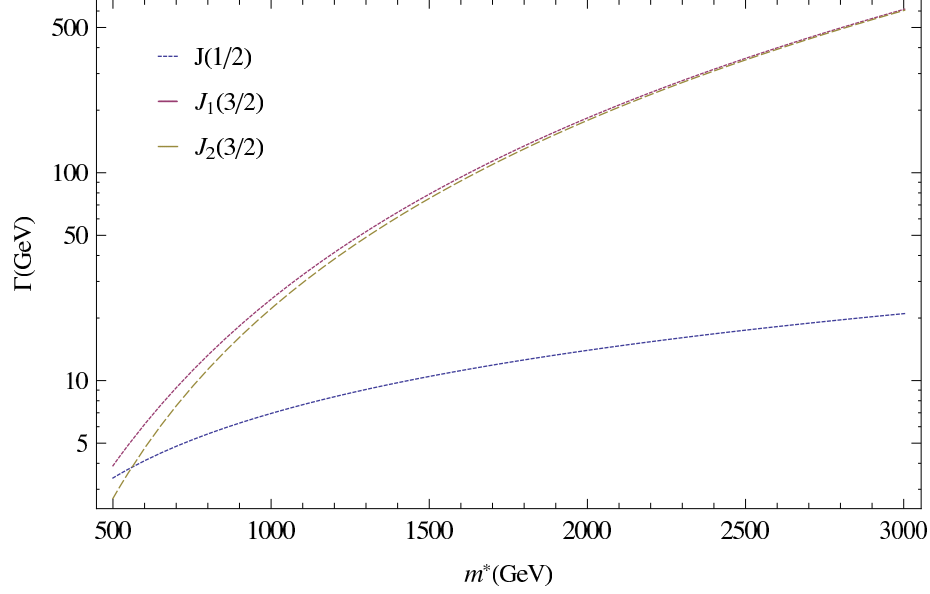
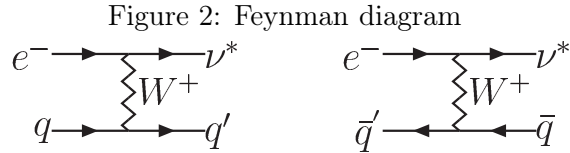


Figure 1: Total decay width of excited neutrinos according to their mass. Here, it is taken $\Lambda = m^*$, $f = -f' = 1$ for excited spin-1/2 neutrinos and $c_{iV} = c_{iA} = 0.5 (i = 1, 2)$ for excited spin-3/2 neutrinos for the two phenomenological currents.

III. SINGLE PRODUCTION AT EP COLLIDER

The excited spin-1/2 and spin-3/2 neutrinos can be produced singly at future ep colliders via t -channel W exchange. In our calculations we use the program CALCHEP [28]. The Feynman diagrams for the subprocess $e^- q \rightarrow \nu^* q'$ and $e^- q' \rightarrow \bar{\nu}^* \bar{q}$ are shown in Figure 2.



Total cross section as a function of excited neutrino mass is shown in Figure 3 for the center of mass energies $\sqrt{s} = 1.3$ TeV and $\sqrt{s} = 1.98$ TeV.

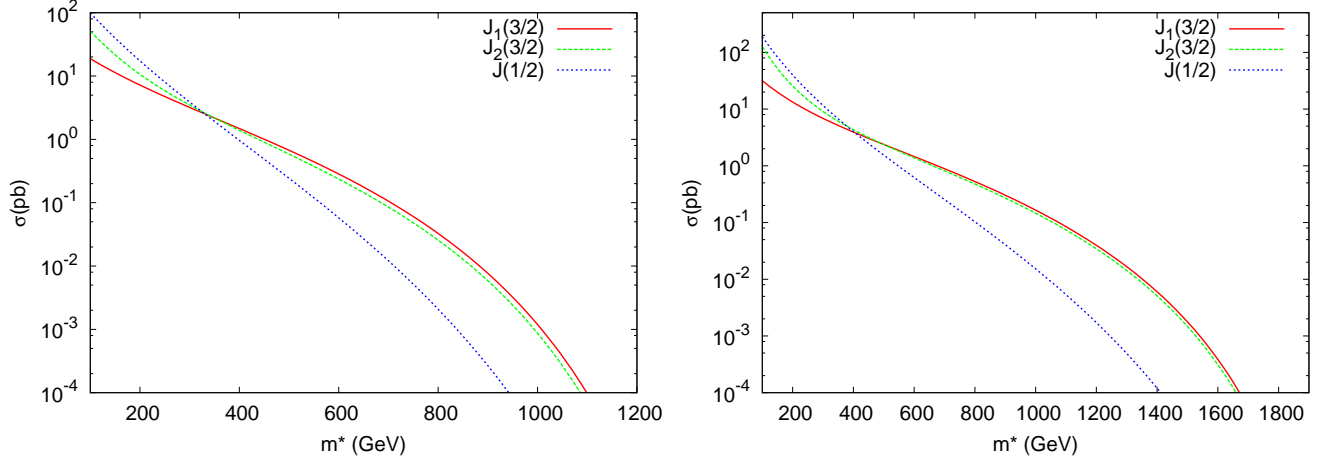


Figure 3: Cross sections for the excited neutrino production with $\Lambda = m^*$ and $f = -f'$ for spin-1/2 ones and $c_{iV} = c_{iA} = 0.5$ ($i = 1, 2$) for spin-3/2 ones at ep collider at $\sqrt{s} = 1.3$ TeV and $\sqrt{s} = 1.98$ TeV.

In our analysis we chose the $\nu^* \rightarrow eW$ mode because of the high branching ratio of the charged current decay channel. We consider the $ep \rightarrow \nu^* X \rightarrow W^+ e^- X$ process and put some kinematical cuts for the final state detectable particles. We deal with the subprocess $e^- q(\bar{q})' \rightarrow W^+ e^- q'(\bar{q})$ and impose the acceptance cuts

$$p_T^{e,q} > 20 \text{ GeV}, \quad (9)$$

$$|\eta^{e,q}| < 2.5 \quad (10)$$

After applying these cuts we obtained the SM background cross section for the process $ep \rightarrow \nu^* X \rightarrow e^- W^+ X$ as $\sigma_B = 0.334$ pb for $\sqrt{s} = 1.3$ TeV and $\sigma_B = 0.928$ pb for $\sqrt{s} = 1.98$ TeV. In order to discriminate the excited neutrino signal we plot the invariant mass distributions for the eW system for the masses $m^* = 400, 500, 600$ GeV at $\sqrt{s} = 1.3$ TeV and for the masses $m^* = 700, 800, 900$ GeV at $\sqrt{s} = 1.98$ TeV in Figure 4 and Figure 5, respectively.

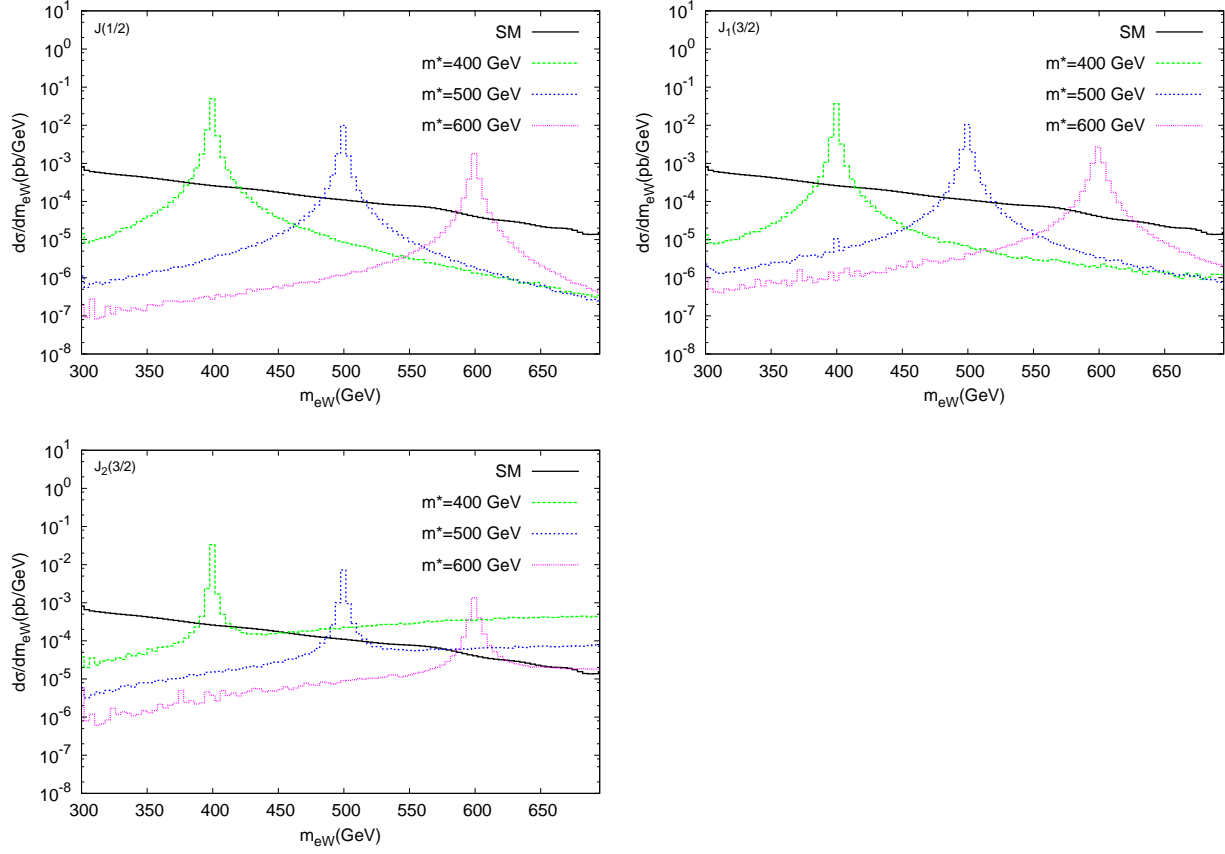


Figure 4: Invariant mass distributions of eW system for the single production of excited spin-1/2 and excited spin-3/2 neutrinos with J_1 and J_2 for $\sqrt{s} = 1.3$ TeV.

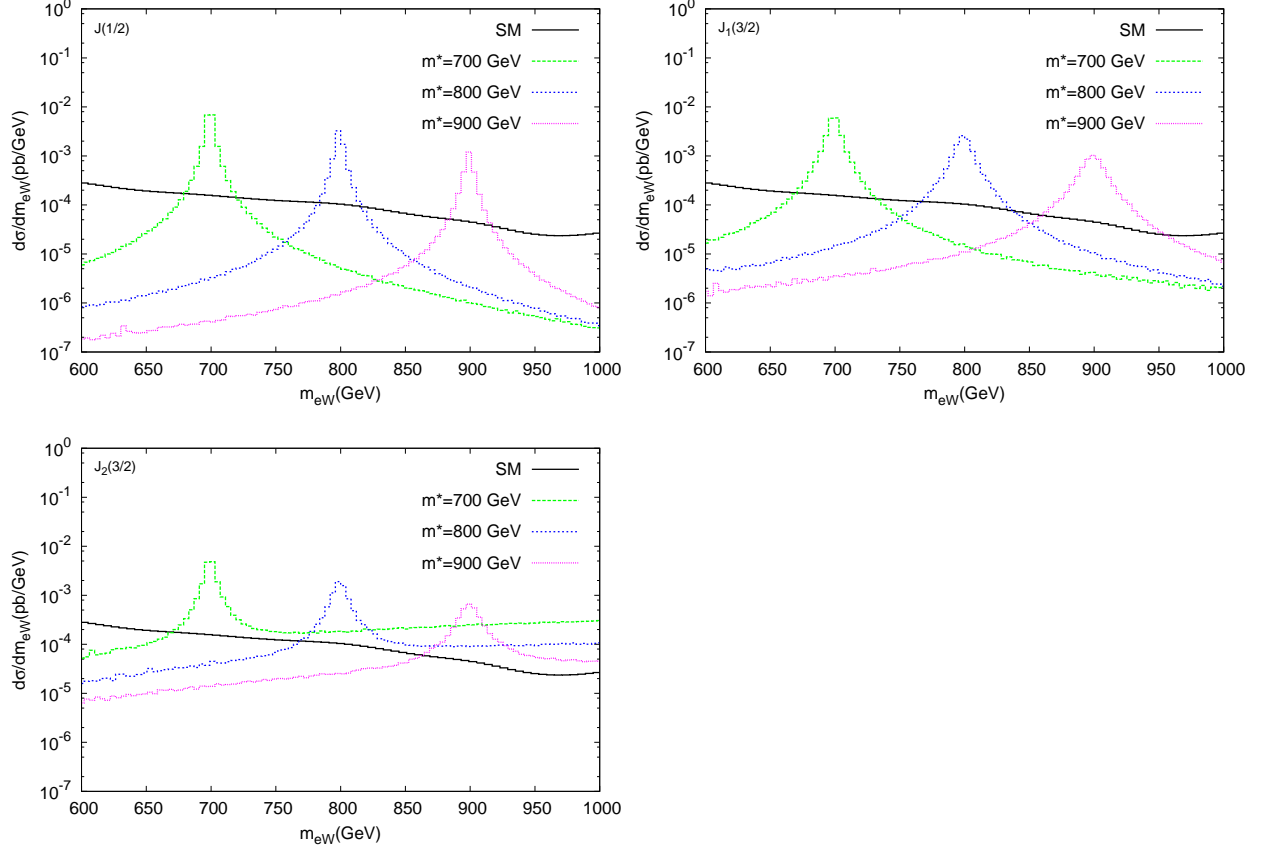


Figure 5: Invariant mass distributions of eW system for the single production of excited spin-1/2 and excited spin-3/2 neutrinos with J_1 and J_2 for $\sqrt{s} = 1.98$ TeV.

We plot the rate of σ_{B+S}/σ_B as a function of excited neutrino mass in Figure 6 to examine the contribution of excited neutrinos to the process $e^-q(\bar{q})' \rightarrow W^+e^-q'(\bar{q})$ and also, to investigate the separation of different excited neutrino models. Here σ_{B+S} corresponds the cross section calculated for the presence of excited neutrino (signal) and Standard Model (background) both, and σ_B is the SM (background) cross section. In these figures, the separation spin-1/2, spin-3/2 with J_1 and spin-3/2 with J_2 excited neutrinos can be easily seen.

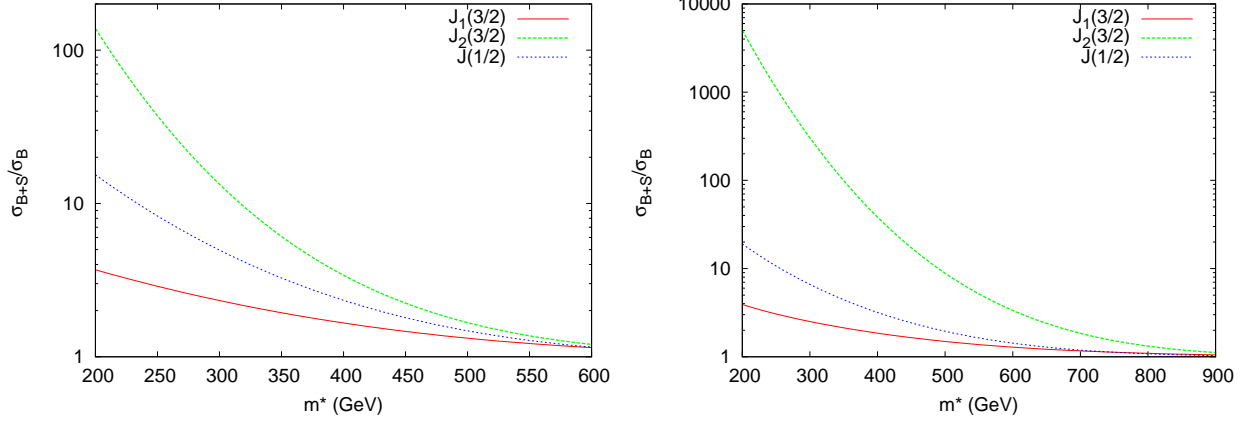


Figure 6: $\sigma_{B+S}/\sigma_B - m^*$ plots for $\sqrt{s} = 1.3$ TeV (left) and $\sqrt{s} = 1.98$ TeV (right).

In order to get accessible limits for the excited neutrinos at high energy ep collider, we plot the contour graphics for excited neutrinos with spin-1/2 and spin-3/2. We choose the W boson decay as $W \rightarrow 2j$. Here we consider the statistical significance

$$SS = \frac{\sigma_S}{\sqrt{\sigma_B}} \sqrt{L_{int}} \quad (11)$$

Here L_{int} is the integrated luminosity of the ep collider and we choose $L_{int} = 100 \text{ fb}^{-1}$ as the LHeC design luminosity. Our results for the SS are shown in Table IV and Table V.

Table IV: Statistical significance SS for ep collider with $\sqrt{s} = 1.3$ TeV for excited spin-1/2 neutrinos and excited spin-3/2 neutrinos with J_1 and J_2 .

$m^*(\text{GeV})$	$SS(J(1/2))$	$SS(J_1(3/2))$	$SS(J_2(3/2))$
400	110.2	75.4	135.6
500	25.5	30.7	30.0
600	5.5	11.9	7.9
700	1.02	4.2	2.2

Concerning the criteria $SS \geq 3$ we plot the $c_{iv} - c_{iA}$ ($i=1,2$) contour plot for excited spin-3/2 neutrinos for the two phenomenological currents and, $f - f'$ contour plot for the excited spin-1/2 neutrinos. In Figure 7 and Figure 8, we choose the excited neutrino mass $m^* = 400$ GeV for the analysis at $\sqrt{s} = 1.3$ TeV and $m^* = 800$ GeV for the analysis at $\sqrt{s} = 1.98$ TeV. We see from these figures the allowed regions for the $c_{iv} - c_{iA}$ ($i = 1, 2$) and

Table V: Statistical significance SS for ep collider with $\sqrt{s} = 1.98$ TeV for excited spin-1/2 neutrinos and excited spin-3/2 neutrinos with J_1 and J_2 .

$m^*(\text{GeV})$	$SS(J(1/2))$	$SS(J_1(3/2))$	$SS(J_2(3/2))$
600	56.3	51.0	235.9
700	22.4	28.0	76.5
800	8.8	15.1	28.9
900	3.3	8.04	12.0
1000	1.2	4.2	5.3

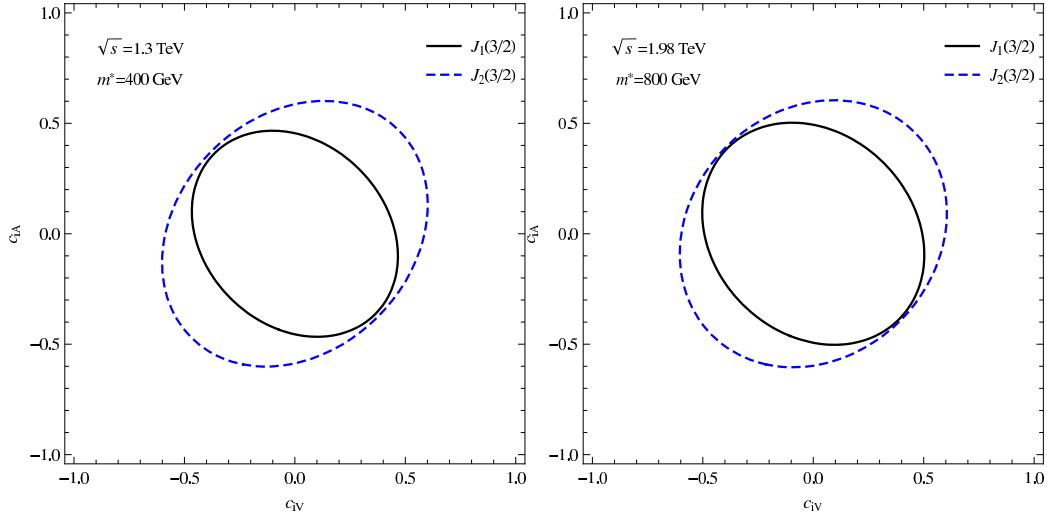


Figure 7: Contour plots for excited spin-3/2 neutrinos for the J_1 and J_2 .

$f - f'$ couplings for the masses $m^* = 400$ GeV at $\sqrt{s} = 1.3$ TeV and $m^* = 800$ GeV at $\sqrt{s} = 1.98$ TeV. The values which we chose in our calculations for the coupling parameters ($c_{iV} = c_{iA} = 0.5$ for the excited spin-3/2 neutrinos and $f = -f' = 1$ for the excited spin-1/2 neutrinos) are compatible with the contour plots.

IV. CONCLUSION

We searched for the excited spin-3/2 neutrino signal at lepton-hadron collider LHeC for two different center of mass energies. We used two different phenomenological current for the spin-3/2 excited neutrinos, and we use the same value of c_{iV}, c_{iA} ($i = 1, 2$) couplings. Since there is no theoretical prediction for the single production of excited neutrinos and

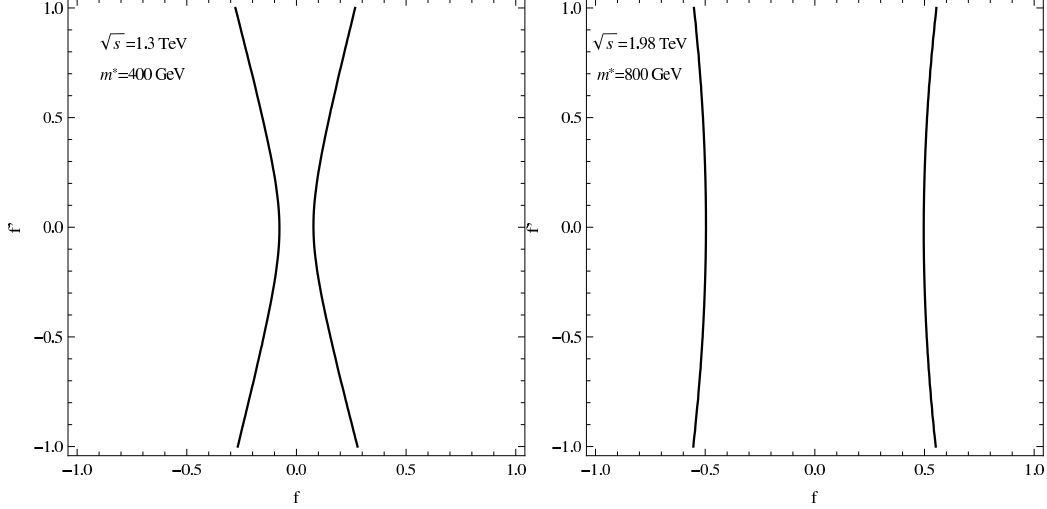


Figure 8: Contour plots for excited spin-1/2 neutrinos.

the effective currents have unknown couplings, we didn't consider the interference between the currents.

In a more detailed calculation one can find an important parameter space in which the interference terms could be important. We also deal with the spin-1/2 excited neutrinos for comparison. Our analysis show that the spin-1/2 and spin3/2 excited neutrino signals discrimination is apparent at next ep colliders. Here we only take into account the effective currents describing the gauge interactions of excited and standard particles. It is possible to include the contact interactions which may enlarge the mass and coupling limits.

Excited neutrinos with different spins would manifest themselves in three families. Here, we only investigate for the excited electron neutrino. It is also possible to make the same analysis for excited muon neutrinos. Single production of excited muon neutrinos is possible at muon-hadron colliders. Physics of μp colliders was studied in [29]. One can find the main parameters of FCC -based μp collider in [30].

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